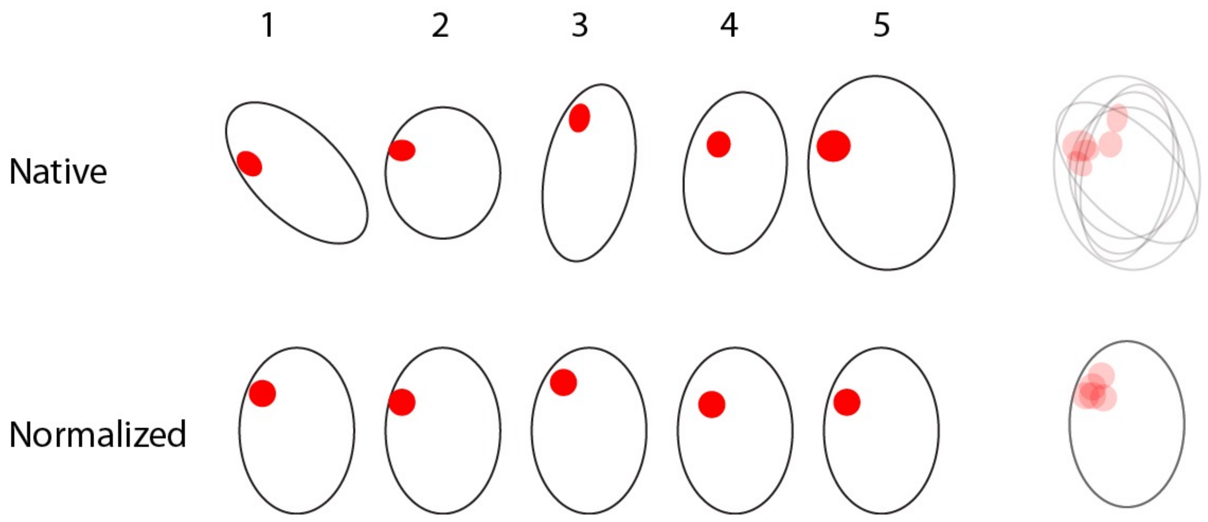
Review on Image Registration

Evolving from classic method to Deep learning



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Deformation Modeling Framework (check supplement for detail)

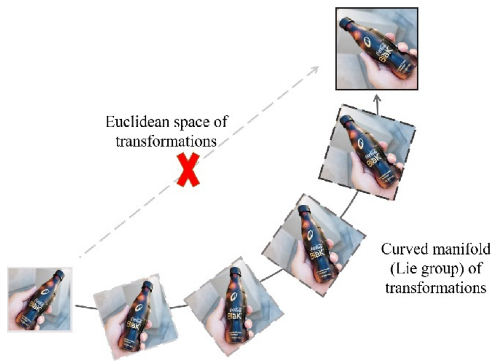
All current models fall in one combination of the following:

- Feature space
  - what data is available?
- Function space
  - what kind of mapping function is assumed?
- Deformation model
  - What is the relationship between Feature space and Function space?
- Similarity measurement
  - how similarly between warpped images and reference images?
- Search strategy:
  - how to improve the accuracy and time/computation expense and manage their trade-off?

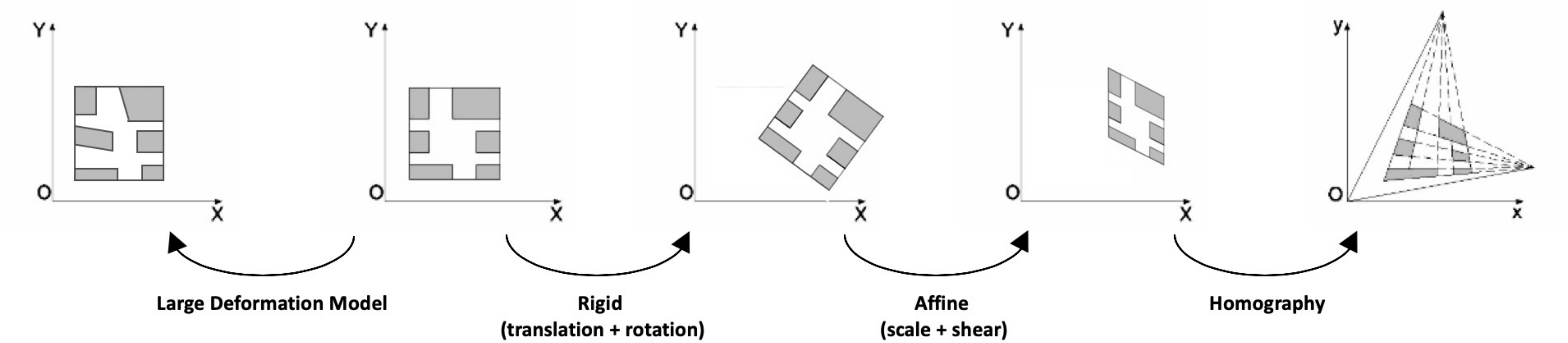
Ideal mapping: Diffeomorphic metric mapping (that is classic and elegant)

The ideal mapping is the **diffeomorphic mapping** that has the key characteristics of Diffeomorphism (preserve topology) listed below. All Image registration algorithms were trying to model in this way!

- smoothness:
  - infinite differentiability: $\frac{\delta\varphi(t)}{\delta t} = v(t)$
  - finite differentiability is more realistic. e.g. twice differentiability is often utilized on of Riemannian manifold
- continuity of mapping:
  - integrablility: $\int_0^a v(t)dt = \varphi(a)$
- invertible derivatives: derivative is continuous
  - invertibility guaranteed: $\varphi_1 \circ I_0 = I_1$ and $\varphi_1^{-1} \circ I_1 = I_0$
  - no folding: Jacobian determinant is non-zero at the point
- smoothness + continuity + invertible derivatives
  - total derivate: all derivatives at a point on the manifold lay on the same tangent space.
- one-to-one:
  - compositions consistency: $(\varphi_2 \circ \varphi_1) \circ x = \varphi_2(\varphi_1(x))$
  - inverse consistency: $\varphi_1 \circ \varphi_{-1} = \varphi_{-1} \circ \varphi_1 = \varphi_0$



Evolution from linear to non-linear mapping



- Algorithm tried to learn the mapping function that is the transformation between source and reference images, denoted as the following:

$$\varphi(x,t) = \frac{\delta I(x,t)}{\delta t}$$

Mapping from I_0 to I_1:

$$\varphi(x,t) \in \mathbb{C}^r$$

Inverse mapping:

$$\varphi^{-1} \in \mathbb{C}^r$$

Identity transform:

$$\phi_0(x) = x$$

- The physical model of deformation evolved from:
  - Elastic Body
  - viscous fluid
  - Diffusion
  - Fluid model (*current*)
- Parameterization (link constraint) became more detailed:
  - Velocity field $\frac{\delta\varphi}{\delta t} = v(\varphi(x,t),t)$
  - Momentum field $v(x,t) = v(x,t) = f(m(x,t),h)$
  - Regularization field $L(x,t) = L(x,t) = f(h(x,t))$

Novel method: Deep learning

- Why move from *traditional methods* to *deep learning*?

Method	Optimization algorithm	Modality of input	Advantage
Traditional method	learn a mapping function for individual source-reference pairs through an iterative process, then warpping	Support multimodal data	Each source image have its "customized" registration
Deep learning	learn a mapping function for all source-reference pair, warpping for each input	currently limited to monomodal data	100X faster during registration than the traditional methods, but usually spend more time during iterative learning

- To speed up registration process
  - Supervised transformation estimation
- To overcome the "golden standard " issue
  - Unsupervised transformation estimation
- To improve the training data size
  - Self-supervise (Contrast) learning

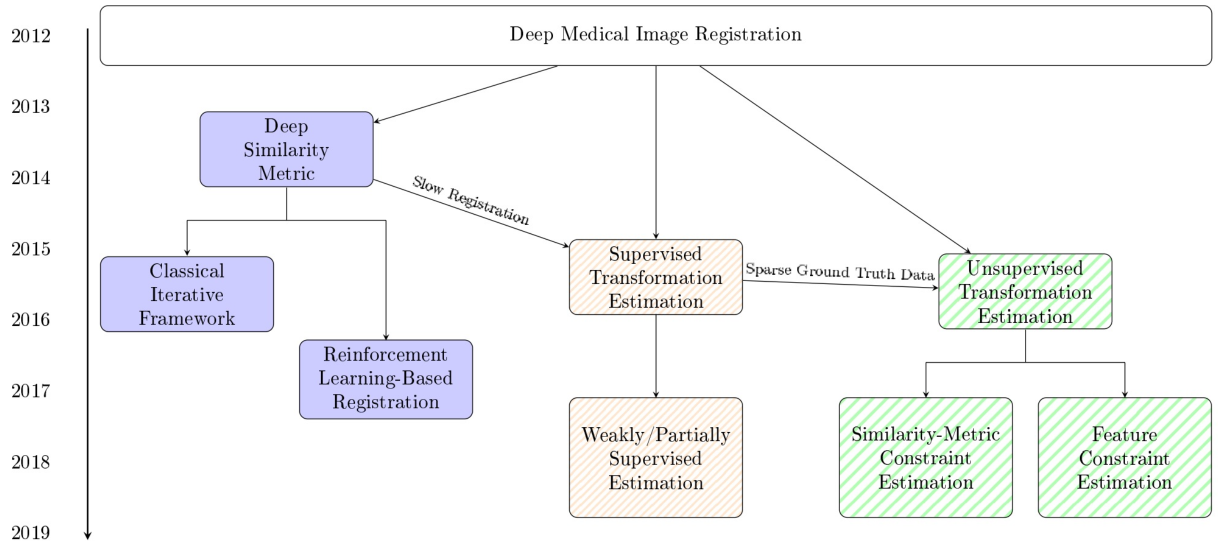


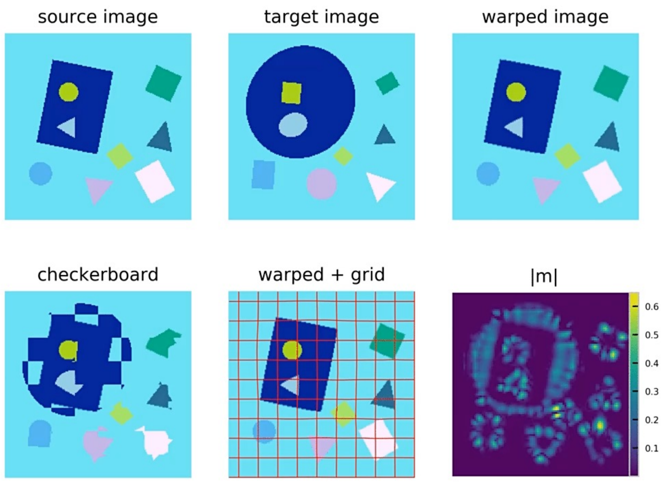
Fig. 1 An overview of deep learning based medical image registration broken down by approach type. The popular research directions are written in bold.

Tackle on the Issue 1: model a smooth mapping

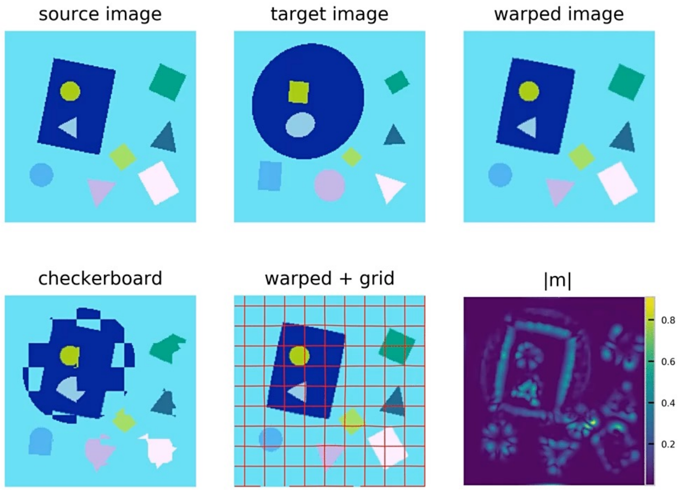
Parameterization of Momentum and Regularization

	Constant Momentum: $v(x,t) = const$	Spatially-varying Momentum: $v(x,t) = v(x) = f(m(x), h)$	Spatio-temporally-varying Momentum: $v(x,t) = v(x,t) = f(m(x,t), h)$
Constant regularizer: $L(x,t) = h = const$	-	Stationary Velocity Field (SVF) e.g DARTEL: $v(x,t) = v(x), L(x,t) = const$	Large Displacement Diffeomorphic Metric mMpping (LDDMM) e.g SHOOT: $v(x,t) = v(x,t), L(x,t) = const$
Spatially-varying regularizer: $L(x,t) = L(x) = f(h(x))$	-	Metric learning for image registration: $v(x,t) = v(x), L(x,t) = L(x)$	-
Spatio-temporally-varying regularizer: $L(x,t) = L(x,t) = f(h(x,t))$	-	-	Region-specific Diffeomorphic Metric Mapping (RDMM), 2019: $v(x,t) = v(x,t), L(x,t) = L(x,t)$

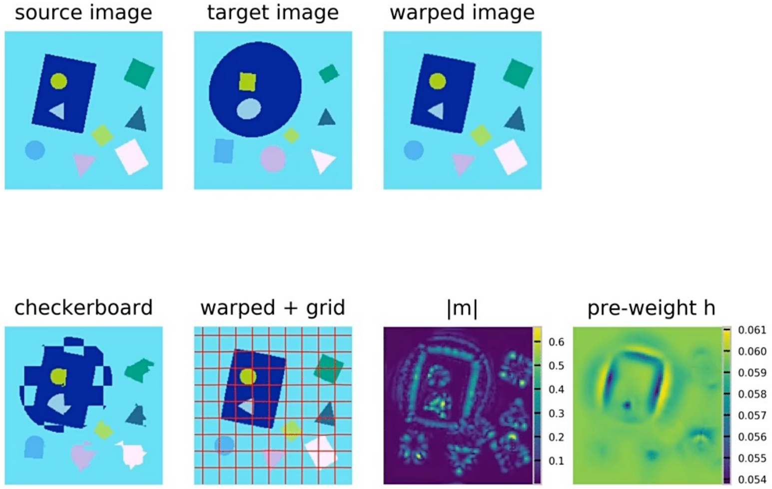
vSVF



LDDMM



RDMM

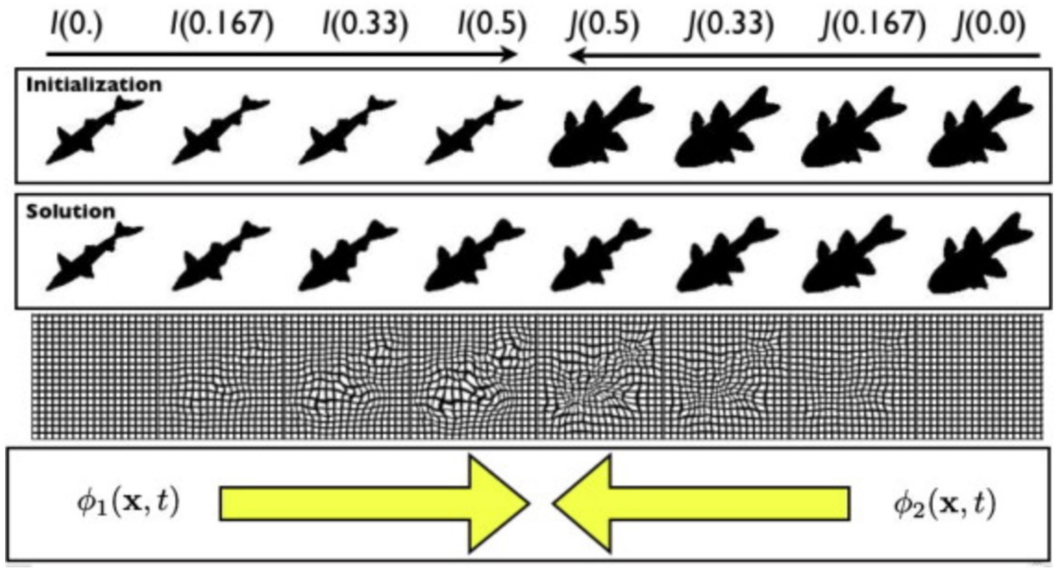


Tackle on the Issue 2: Absence of “Gold Standard”

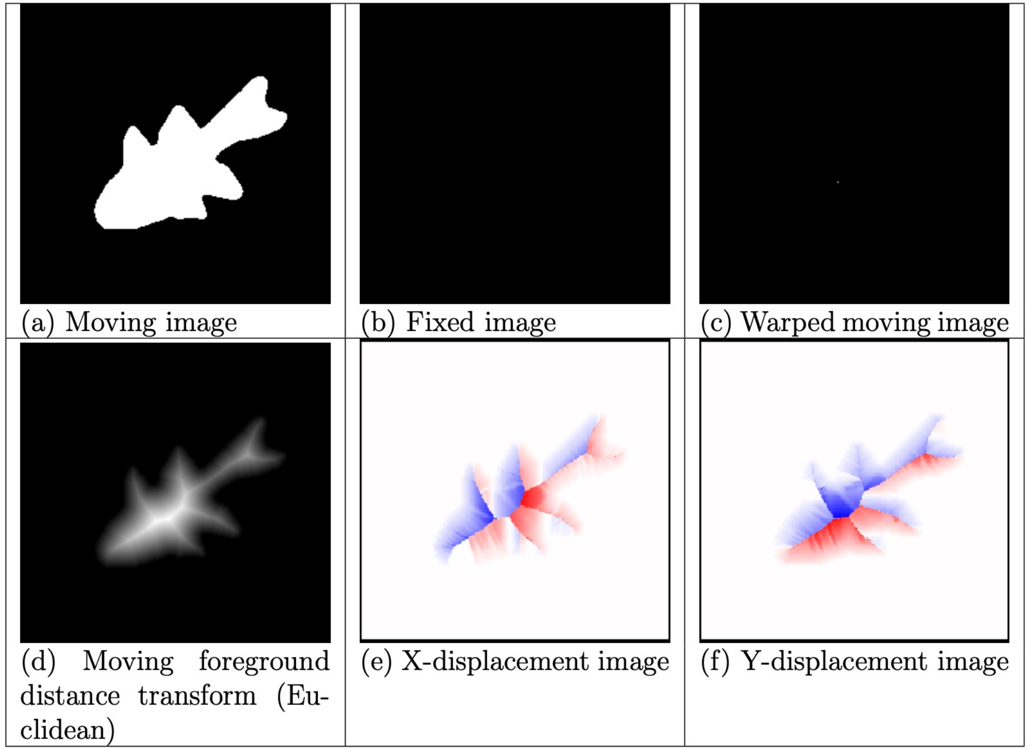
- Computer-generated scenes, self-supervised learning
  - Apply inverse warping and noise to the template to generate non-registered images.
  - Evaluate the algorithm performance by experimenting on different modifications of warping and noise

Tackle on the Issue 3: Invertibility unguaranteed

- Found solution: symmetrical optimization (ANTs::SyN), symmetry loss
  - Optimized the parameter of mapping function from both way, e.g. source->reference and reference->source.



Tackle on the Issue 4: Shape collapse problem



(a) Moving image
(b) Fixed image
(c) Warped moving image
(d) Moving foreground distance transform (Euclidean)
(e) X-displacement image
(f) Y-displacement image

- Not yet found a feasible solution from the literature

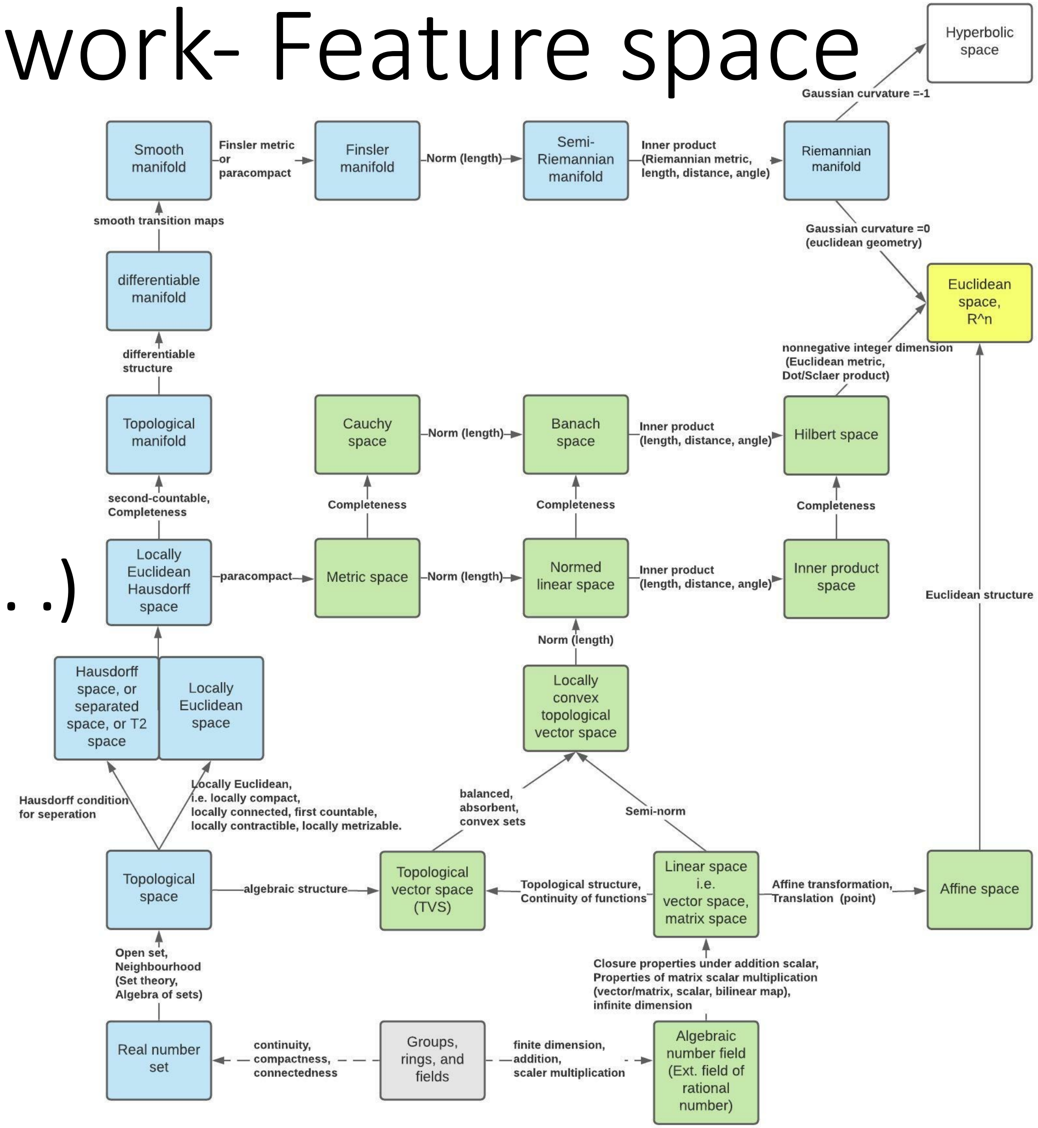
Next to investigate:

- General
  - What happens during learning in the flow vector field and Jacobean field? (both the end and middle result)
  - How to target the difficult region at the early epoch?
- ABC-DS specific
  - How do shrinking and expansion cause worse issues for ABCDS data?
  - Is it possible to create an ABC-DS template through dimensional reduction with unsupervised deep learning?
- Deep learning specific
  - Is it possible to do embedding?
  - What is the appropriate size of hyper-parameter for the ABC-DS task?

Supplement: Modeling Framework- Feature space

How is the data?

- distance measure
- dimensionality of images (d = 2, 3, 4, . . .)
- modality (binary, gray, color, . . .)
- mono-/multimodal images
- acquisition (PET, MRI, CT, SPECT . . .)
- inter/intra patient

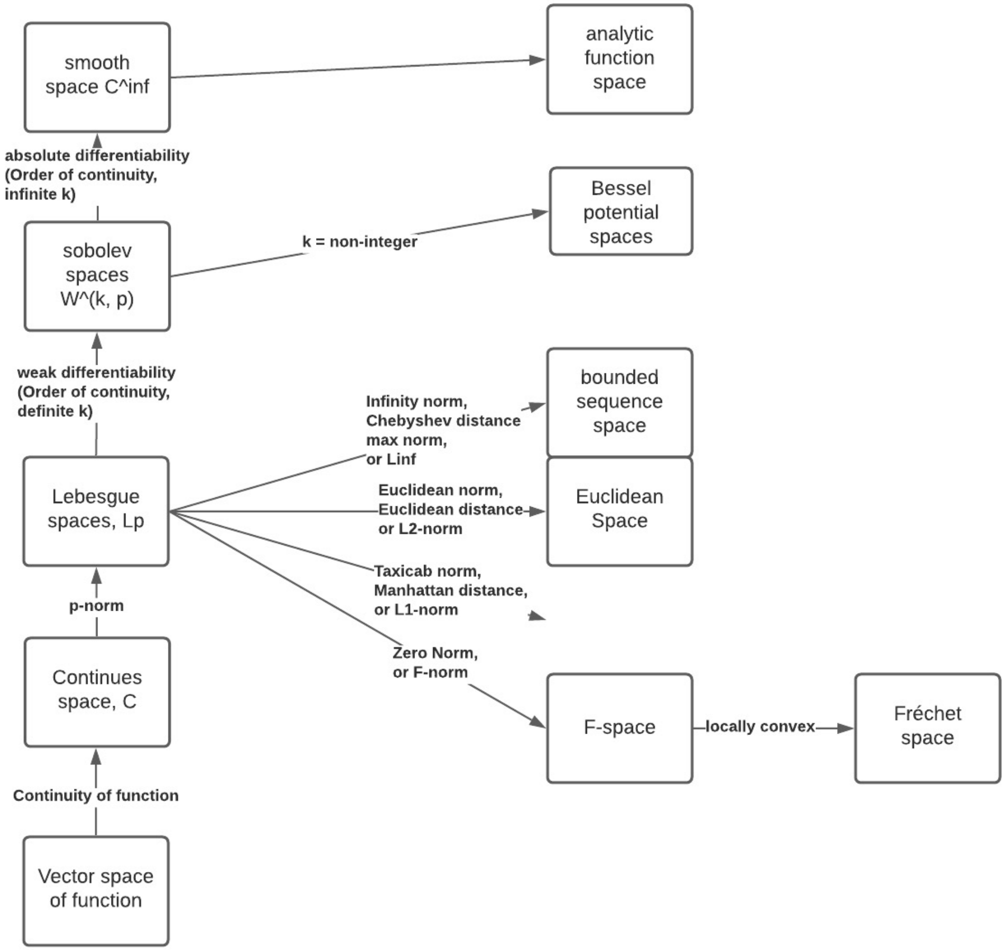


Construction of mathematical space
(not reviewed yet, please don't quote)

Supplement: Modeling Framework- Function space

What kind of mapping function is assumed?

- Sobolov spaces $\mathbb{W}^{k,p}$
- smooth space $\mathbb{C}^{\infty}$



Construction of Function Space
(not reviewed yet, please don't quote)

Supplement: Modeling Framework- Deformation model

What is the relationship between Feature space and Function space?

- linear approach
    - rigid (translation, rotation, reflection),
    - affine (translation, rotation, scaling, reflection, shearing),
- non-linear approach
    - elastic registration
    - viscous fluid flow models
    - diffusion registration
    - curvature registration
    - fluid registration (SHOOT, DARTEL, deep learning)

Supplement: Modeling Framework- Image similarity

How similarly is measured between warped images and reference images?

- Landmarks-based
    - sum of squared difference
- Image-based
    - mutual information
- Point-based
    - normalized cross correlation
    - sum of squared difference
    - Optical/Scene Flow

Supplement: Modeling Framework- Search strategy

How to improve the accuracy and time/computation expense and manage their trade-off?

- Numerical method
    - Gradient descent method
    - Finite Element method
    - Newton–Raphson method
    - Powell method
    - Genetic Algorithm

- Regularization
    - local linearization
    - global linearization + velocity field interpolation
    - Diffusion Regularization
    - Curvature Regularization
    - Bending Energy
    - Linear Elasticity
    - Volume Preservation
    - folding prevention
    - smoothness